hadronic component of cosmic rays and attenuated the muon flux by \( \approx 2 \cdot 10^3 \). Extreme care was taken to choose materials with low levels of radioactivity and to avoid contamination of the germanium itself.

A.3 Data Analysis

The data set used for this analysis was taken from the second detector from April to November of 1988. It consists of 90.3 kg-days of live time. Figure A.1 shows the energy spectrum measured by this detector. The detector has a threshold of 2.6 keV (a conservative number—where the measured rate is under 20 counts/kg-keV-d). The large peak at around 10 keV is actually two peaks from \(^{68}\text{Ga}\) and \(^{68}\text{Cu}\) decays which originate from spallation products in the germanium. However, it does not affect the exclusion limits and actually serves as a convenient energy calibration point.

The first filtering of the data takes place on an event by event basis. Any event that is coincident with a signal in the NaI veto or the other detector is eliminated. Next, any event that is less than or equal to one second after a previous event is rejected as a noise burst. An event is also rejected if the pulse shape is sufficiently bad (the signal is measured in two different electronics channels for each detector). These cuts eliminate much of the microphonics in the low energy range at the price of a small reduction in the live time.

The second stage of the filtering consists of cuts made on larger time periods of data (roughly 1-2 hours). The goal of this is to eliminate especially noisy periods of data collection. Cuts on the number of events, the number of noise bursts, and the number of bad pulse shapes result in a 30.0% dead time. Each of these distributions is consistent with a Poisson distribution of expected events and a sometimes large tail, which is interpreted as noise or problems with the electronics.

Energy calibration of the data is periodically checked. The \(^{68}\text{Ga}\) peak at 10.3 keV and the \(^{214}\text{Pb}\) peak at 351.9 keV can be reliably used. No significant drift of these was observed.
Figure A.1: Measured energy spectrum for the second detector at Oroville. The counting time was 90.3 kg-d.

Next, to set exclusion limits, a theoretical model of the expected dark matter rate is needed. The halo is assumed to be made up of only one type of particle; no hot dark matter is considered. The velocity distribution is assumed to be an isotropic Maxwell-Boltzmann distribution truncated at $v_{esc}$ and the earth's velocity is then vectorially added with a random angle. The rate is corrected for the ionization efficiency of germanium [66] and also for the loss of coherence by adding a form factor, $\exp\left(-\frac{E_{esc}}{E_{coh}}\right)$. The relevant parameters are

\begin{align}
\rho_{halo} &= 0.3 \text{ GeV/cm}^3 \\
v_{rms} &= 261 \text{ km/s} \\
v_{esc} &= 640 \text{ km/s} \\
E_{coh} &= 53.76 \text{ keV}. \tag{A.1}
\end{align}

Three different analysis methods were utilized. These vary in their degree
of conservatism, and the resulting limits reflect this (although there is not a large difference between any of them—obviously, the most important factor is the detector itself). The energy range examined is from a conservative detector threshold of 2.6 keV up to 40.0 keV.

A.3.1 Method I

This analysis uses only those energy bins where the calculated dark matter rate exceeds the measured counting rate, $N_{th} > N_{obs}$. A quantity, $\kappa^2$, resembling a chi squared but not a true chi squared, is calculated using only these selected bins. No background subtraction or fitting is used in this method.

A Monte Carlo calculation is needed to determine the 90% confidence level for $\kappa^2$. The procedure is to use the appropriate Gaussian or Poisson statistics to fluctuate the number of counts in each bin predicted by the theory. These bins are compared to the number of observed counts and a $\kappa^2$ is formed. $\kappa^2 = 0$ if none of the bins have $N_{th} > N_{obs}$. As the cross section is increased, more of the theory bins exceed the measured counts and a positive $\kappa^2$ results. The distribution of $\kappa^2$ as a function of cross section is calculated and the 90% confidence level is determined as the cross section for which only 10% of the $\kappa^2 = 0$. The next section provides further explanation of this procedure.

A.3.2 Method II

This method also uses no background subtraction. The criteria here is that three consecutive energy bins are found where the measured counts are 1.2 standard deviations below the theoretical prediction. This is interpreted as a 90% confidence level. However, a Monte Carlo calculation is again needed to verify this.

This analysis is perhaps not as sensitive as method I. The data could conspire so that finding three consecutive bins that satisfy this method results in a more conservative cross section than method I would give. This technique is also probably more dependent on bin size; the larger the bins in energy, the more
A conservative the resulting cross section is.

A.3.3 Method III

This analysis consists of fitting the background shape and then adding dark matter signal while allowing the parameters of the fit to vary. A $\chi^2$ is then formed to find the 90% confidence level. Because a background subtraction is involved, the limits from this approach will be the least conservative.

For this detector, the following were fitted: two Gaussians for the $^{68}$Ga and $^{68}$Cu peaks, an exponential decay starting at the detector threshold, and a flat background to account for accidentals at higher energies. This sort of analysis assumes that the backgrounds are very well understood. Another problem is that this fit does not go to zero at zero energy, which represents an unphysical situation. Trying to characterize fully the detector noise at threshold is a much more difficult problem, however.

A.4 Comments on the Monte Carlo Procedure

To calculate a $\chi^2$ for a counting experiment with bins having few events, the Review of Particles Properties [6] recommends using:

$$\chi^2 = \sum_i \left[ 2 \left( N_{th}^i - N_{obs}^i \right) + 2 N_{obs}^i \ln \left( \frac{N_{obs}^i}{N_{th}^i} \right) \right]$$ (A.2)

where $N_{th}$ is the number of counts in a bin predicted by theory and $N_{obs}$ is the number of observed or measured counts in the bin. This equation converges to the usual Gaussian case for higher numbers of counts and yields correct error estimates for all $N_{th}$.

In method I, all bins are not summed over; only bins where $N_{th} > N_{obs}$ are used. Therefore, the quantity being calculated, $\kappa^2$, resembles but is not a true $\chi^2$. A Monte Carlo calculation is then needed to determine confidence levels of such a quantity. To do this, one has to decide whether to fluctuate the actual data and compare this to theory or to fluctuate the theory prediction
and compare this to the data. Again, the Review of Particle Properties states, "For small \( n_0 \) [or \( n_{\text{obs}} \)] we can define an upper limit \( N \) for \( \mu \) as being that value of \( \mu \) such that it would be at least \( 1 - \alpha \) (e.g. 90\% or 95\%) probable that a random observation of \( n \) would then lie above the observed \( n_0 \)." Thus,

\[
1 - \alpha = \sum_{n=n_0+1}^{\infty} f(n; N); \quad \alpha = \sum_{n=0}^{n_0} f(n; N). \tag{A.3}
\]

In an experiment such as this, only \( n_0 \) is known, not \( \mu \). \( f(n; \mu) \) must be inverted using Bayes' theorem to find \( f(\mu; n) \):

\[
df(\mu; n_0) = \frac{f(n_0; \mu)f(\mu)d\mu}{\int_0^{\infty} f(n_0; \mu)f(\mu)d\mu}. \tag{A.4}
\]

Then, the 90\% confidence level on \( \mu \) is the value of \( \mu_0 \) for which there is a 90\% probability of \( \mu < \mu_0 \) is

\[
0.90 = \int_0^{\mu_0} df(\mu; n_0). \tag{A.5}
\]

If the data is fluctuated, \( f(n; n_0) \) is calculated using appropriate statistics and \( N \) is excluded if

\[
0.90 = \sum_{n<N} f(n; n_0). \tag{A.6}
\]

On the other hand, if the theory is fluctuated, the probability of finding greater than \( n_0 \) counts given \( N \) is being found. The calculation is

\[
0.90 = \sum_{n>n_0} f(n; N), \tag{A.7}
\]

which is equal to \( 1 - \alpha \) as seen in equation A.3. This appears to be the correct way to determine the confidence level. In practice, both approaches give similar results if the number of counts in each bin is large enough. However, in the case where there are no counts in a bin, there is indeed a distinct difference which should invalidate the case for fluctuating the data [67].

A.5 Results

The three excluded regions for the Oroville data set are shown in figure A.2. As predicted, method II is the most conservative. Method I shows roughly
Figure A.2: The different 90% confidence level exclusion limits for the three types of analysis. The data set is 90.3 kg-d taken at Oroville.

A factor of two improvement over method II over the range of dark matter masses. Method III gives the best limits for low mass due to the background subtraction near the detector threshold and then is comparable to method II at higher masses where the subtraction is not so large an effect. The bump at the minimum in the limits for method III is the result of the fit behavior near the detector threshold. If the fit is bad enough initially, adding dark matter will help the $\chi^2$. As the fit improves, adding signal does not help as much. What is desired is an initially flat $\chi^2(\sigma)$ function which then rises as $\sigma$ is increased.

A variation on method III would be to fit only the known radioactivity peaks and their tails. One would not fit the noise near the detector threshold nor the higher energy spectrum above 10 keV. This approach would allow one to be more confident that the fit is well understood, but the limits would probably
not be any better than the other two methods.

A.5.1 Ge Exclusion Limits

The exclusion limits from the Oroville experiment should be compared to those of other fairly recent experiments. Figure A.3 shows Oroville's measured energy spectrum compared to three other germanium dark matter experiments: an experiment at Gotthard [68], the Heidelberg-Moscow experiment [69], and an experiment at the Canfranc tunnel [70]. To check their published exclusion limits, the relevant parameters in the theoretical model (see equation A.1) were changed to agree with what the other groups used.

1993 data from the Heidelberg-Moscow experiment was checked using method II. Their data set consisted of 165.6 kg-d of counting with enriched $^{76}$Ge. The detector threshold was 11 keV due to its large mass of 2.76 kg. The higher threshold results in worse exclusion limits at low mass, but their lower backgrounds produce the best limits for masses above 40 GeV. The reanalyzed exclusion limits were identical to what was published in their paper.

Another check was done using data from the Canfranc experiment. They counted with natural Germanium for 85.2 kg-d with a threshold of 1.6 keV. Their lower threshold produced the best exclusion limits for masses below roughly 15 GeV. Again, the reanalyzed limits correspond fairly well with what that group has claimed. Slight discrepancies are probably explained by a difference in the theoretical model used to calculate the dark matter rate.

The third experiment was done at Gotthard. Their data set consists of 51.6 kg-d of counting time with natural germanium with a threshold of 1.8 keV. As seen in figure A.3, their background worse than that of Oroville's except very near their threshold (the Ga peak is inconsequential). However, the exclusion limits that were published [68] were roughly an order of magnitude better than that of Oroville's limits! To further explore this, figure A.4 shows Gotthard's background versus the calculated dark matter rate for a given mass and cross section that they “excluded” at the 90% confidence level.
Figure A.3: Comparison of the backgrounds at the Gotthard, Heidelberg-Moscow, and Canfranc experiments with the background at Oroville. Oroville is the solid histogram in each case. Note that the vertical scales differ.
**Figure A.4:** *Comparison of the Gotthard background with a theoretical dark matter rate for \( m = 18 \text{ GeV/c}^2 \) and \( \sigma = 3.6\times10^{-35} \text{ cm}^2 \) that they "excluded" at the 90% confidence level.*

The purpose of this comparison was twofold. First, it resolves the discrepancy with the published extraordinary Gotthard limits and their ordinary measured background. They strongly disagree with the limits obtained from three other similar experiments. It is most probably a miscalculation in their dark matter rate or simply a problem with their analysis. Secondly, and more importantly, this comparison presents limits from four different experiments using the same analysis technique and same theoretical model for calculating the dark matter rate (the coherence energy was changed to reflect different detector masses). Figure A.5 shows the 90% confidence level exclusion limits obtained from method II for these four experiments.
Figure A.5: The 90% confidence level exclusion limits for Ge experiments. The analysis is the same in all cases, method II, and the theoretical parameters are also identical.

A.5.2 SIMP Exclusion Limits

The main thrust of the search for dark matter has been limited to weakly interacting cross sections. Exclusion limits can also be placed on strongly interacting dark matter candidates (SIMPs). These are generally particles that have higher masses and cross sections than WIMPs and there are still several large regions of parameter space that have yet to be excluded. Starkman et al. [71] provides the background for this idea. Essentially, the shielding and overburden, high Z materials, for typical low background experiments degrade the SIMP energy since the cross section goes roughly as $A^4$.

Data were taken at Oroville for 5 days with the Pb lid and the NaI veto counters removed. The experiment was only at 600 m.w.e., so it is more sensitive